Optimal RWA for Static Traffic in Transmission-Impaired Wavelength-Routed Networks

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Abstract—Physical-layer degradations can severely limit the capacity of all-optical wavelength-routed networks, and powerful routing and wavelength assignment (RWA) algorithms are needed to minimize their effect. An optimal RWA technique is presented that minimizes the number of wavelengths needed to serve a set of static call provisioning requests. The optimal solution is compared to classical suboptimal algorithms and results show that route selection is more critical than wavelength assignment if multiple paths are available. Physical impairments become less significant in mesh networks than in tandem networks.

Index Terms—Wavelength division multiplexing, networks, crosstalk, routing and wavelength assignment.

I. INTRODUCTION

Optical networks are becoming core components of metro and regional information infrastructures. Replacing electrical switches with all-optical switches promises a more cost effective and flexible optical network design solution. Avoiding optical-electrical-optical conversion has one potential problem: lightpaths in the networks can be many hundreds of kilometers long with no regeneration other than optical amplification and dispersion compensation. Linear and nonlinear distortion and crosstalk can accumulate causing so much physical layer degradation as to make the lightpath unusable. Avoiding this physical-layer degradation is a major challenge facing the optical network research community.

Wavelength switched optical networks require routing and wavelength selection algorithms to establish a lightpath for each call request. There has been a flurry of recent research activity aimed at designing powerful routing and wavelength assignment (RWA) algorithms that respond to the instantaneous network state and minimize the effects of the physical layer impairments [1]–[4]. These so-called quality-of-transmission (QoT) aware algorithms are ad-hoc designs whose performances are compared via simulation to other suboptimal solutions for specific network topologies.

This paper presents an optimal routing and wavelength assignment algorithm that satisfies given QoT constraints for a static lightpath provisioning environment [4]. The optimality criterion used is the number of consecutive wavelengths needed to support the call requests. The goal in describing the optimal solution is not to present a viable algorithm (this is impractical for larger networks given the NP-hard nature of the problem), rather to provide a framework by which the relative importance of various effects can be quantified. Optimal algorithms have been previously proposed for networks suffering no transmission impairments [5], [6] and for systems affected only by a single physical degradation [2]. With our technique the total effect of the physical layer on the routing and the wavelength assignment problem can be isolated since artifacts based on the choice of RWA have been removed. Results for three different networks comparing the optimal solution to a standard first-fit/shortest path technique show the maximum gain that can be obtained by any QoT-aware design.

II. PROBLEM FORMULATION

We consider the problem of establishing a given set of requests in an optical network with non-negligible physical impairments using the least bandwidth. If the physical degradations are ignored, this static RWA problem can be solved using integer linear programming (ILP), based on either a flow model (link states as variables) or a route model (lightpaths as variables) [5], [6]. The route formulation is employed here because it is reputed to have better performance [5], [6]. We add to this formulation a QoT constraint equivalent to a maximum acceptable bit error rate (BER). The problem has to be restated because with physical distortion the wavelengths can no longer be treated identically. In the resulting optimization problem, neither the cost function nor the constraint set remains linear.

We propose a new non-linear programming formulation by extending the models in [5], [6], using the same notation for the sake of consistency.

Let $G(N, V)$ be the network graph consisting of $N$ nodes, $N = \{1, 2, \ldots, N\}$ and $L$ vectors (links), $V = \{1, 2, \ldots, L\}$. Assume that each bidirectional fiber is represented by two unidirectional links (the model can be easily extended to the multi-fiber case). A path $p \subseteq V$ is a connected sequence of links excluding any cycle. Define $I(j \in p) = 1$ if $j$ is a link of path $p$ and $I(j \in p) = 0$ otherwise. $Z$ is the set of all possible source-destination (s-d) pairs and $\Lambda$ is the static deterministic traffic matrix, in which $\Lambda_z$ denotes the number of connections needed between s-d pair $z \in Z$. Let $R$ represent the number of connections requested. Define $A_z = \{p : s \to d\}$ to be the set of routes for s-d pair $z$.

A lightpath is identified by the path $p$ and its wavelength $i$. Wavelengths in each fiber are numbered sequentially. No wavelength conversion is assumed as these devices are still in experimental phases of development. Define $\delta_{p,i}^z = 1$ if $(p, i)$ is selected as an active good-quality lightpath for $z$ and
\(\delta_{z,p,i} = 0\) otherwise. Let \(\lambda_i = 1\) if wavelength \(i\) is used by at least one lightpath in the network, equal 0 otherwise.

In [5], [6], the cost function is simply the total number of wavelengths used. Under the same criterion, when transmission impairments are considered the algorithm would choose nonadjacent wavelengths since they experience less degradation. We seek to minimize the network cost by minimizing the total continuous bandwidth needed; therefore, we must minimize the number of consecutive wavelengths used instead of the total number of wavelengths occupied. This makes the objective function nonlinear. The problem can be written as

\[
\min \quad \max_{1 \leq R, 1 \leq i \leq t} \lambda_i \lambda_j \vert i - j + 1 \vert \quad (1)
\]

subject to:

\[
\sum_{i=1}^{R} \sum_{p \in A_z} \delta_{z,p,i} = A_z, \quad \forall z \in Z \quad (2)
\]

\[
\lambda_i \geq \delta_{z,p,i}, \quad \forall z \in Z, \forall p \in A_z, \forall 1 \leq i \leq R, \quad (3)
\]

\[
\sum_{z \in Z} \sum_{p \in A_z} \delta_{z,p,i} I(j \in p) \leq 1, \quad \forall j \in V, \forall 1 \leq i \leq R, \quad (4)
\]

\[
C_{p,i,z} \leq C_{th}, \quad \forall z \in Z, \forall p \in A_z, \forall 1 \leq i \leq R, \quad (5)
\]

where the objective function (1) is the total continuous bandwidth needed in units of wavelength channels and (5) represents the QoT constraints. \(C_{p,i,z}\) is a function of the impact to QoT from physical impairments, and \(C_{th}\) is the threshold.

The above objective function can be approximated by a quadratic form,

\[
\min \sum_{i} \sum_{j \neq i} \lambda_i \lambda_j (i - j + 1)^q, \quad (6)
\]

where \(q\) is a large even number. The problem can then be formulated as a quadratically constrained quadratic program (QCQP) if \(C_{p,i,z}\) is quadratic, which is true in many practical cases. Approximation methods can then be used to efficiently optimize wavelength usage for large networks.

**III. QoT CONSTRAINT**

In this section we show how the QoT constraint in (5) can be formulated in a quadratic form considering several important physical degradations. We include noise, such as amplified spontaneous emission (ASE) noise from the optical amplifiers, shot noise, and thermal noise at the receivers, and node crosstalk, from power leaks in the optical crossconnects (OXC) and from adjacent channels due to imperfect WDM demultiplexing. The adjacent channel crosstalk can be further categorized as self-crosstalk and neighbor-crosstalk [7].

Assuming binary on-off keying (OOK) modulation, the total noise variance at the receiver in lightpath \((p, i)\) for s-d pair \(z\) for bit \(k \in \{0, 1\}\) can be written as

\[
\sigma^2(k) = \sigma^2_{X,T}(k) + \sigma^2_{ASE}(k) + \sigma^2_{th} + \sigma^2_{sh}(k), \quad (6)
\]

where \(\sigma^2_{ASE}(k)\), \(\sigma^2_{th}(k)\), and \(\sigma^2_{sh}(k)\) are variances for bit \(k\) due to ASE noise, thermal noise, and shot noise, respectively, computed as in (6.1.7), (4.4.5), and (4.4.8) of [8].

The crosstalk variance \(\sigma^2_{X,T}(k)\) depends on the variables \(\delta_{z,p,i}\). We define \(K_{p,i,z}\) as the number of switching fabric crosstalk terms induced by path \(p\) on \(p\) (same wavelength) and define \(K_{p,i,z}^{self}\) as the number of self-crosstalk terms in path \(p\) induced from adjacent channels in \(p\) and \(K_{p,i,z}^{self}\) represent the total number of switching fabric and adjacent-channel self-crosstalk terms experienced by lightpath \((p, i)\) for s-d pair \(z\), respectively, as a function of the network state. Both yield quadratic functions that can be easily counted by the method presented in [4],

\[
N_{z,p,i}^{sw} = \sum_{z \in Z} \sum_{p \in A_z} \delta_{z,p,i} \delta_{z,p,i} K_{p,i,z}^{sw}, \quad (7)
\]

\[
N_{z,p,i}^{self} = \sum_{z \in Z} \sum_{p \in A_z} \delta_{z,p,i} \delta_{z,p,i}^{self} K_{p,i,z}^{self}, \quad (8)
\]

The neighbor-crosstalk is a cubic term that has to be computed hop by hop. Assume that \(p\) and \(p_1\) share a set of hops, \(1, \ldots, n’\). Then the neighbor-crosstalk term is given by

\[
N_{z,p,i}^{nbr} = \sum_{m=1}^{n'} \sum_{p \in A_z, p_2 \in A_z, p_1 \neq p_2 \neq p} \delta_{z,p,i} \delta_{z,p,i} \tau_{m,p,p_2} \quad (9)
\]

where \(\tau_{m,p,p_2}\) is 1 if \(p_1\) and \(p_2\) share the same input demultiplexer and \(p\) and \(p_1\) share the same output demultiplexer at hop \(m\). If the problem must be formulated as a QCQP, this crosstalk can be ignored, as it generates much fewer terms than (7) or (8).

The total crosstalk variance in lightpath \((p, i)\) for s-d pair \(z\) is calculated by [1]

\[
\sigma_{X,T}^2(k) = \rho^2 P_o(k) \left[ X_{sw} N_{z,p,i}^{sw} + X_{adj} N_{z,p,i}^{self} + N_{z,p,i}^{nbr} \right],
\]

where \(\rho\) is the receiver responsivity, \(P_o(k)\) is the received optical signal power if bit \(k\) is sent. \(X_{sw}\) and \(X_{adj}\) denote the switching fabric and adjacent-channel crosstalk power level attenuations with respect to the main signal, respectively.

Usually QoT is measured using the Q-factor, given by

\[
Q = \frac{P_o(1) - P_o(0)}{\sigma(1) + \sigma(0)} \quad [8].
\]

Since only the denominator depends on the instantaneous network state, we write constraint (5) simply as

\[
C_{p,i,z} = \sigma(1) + \sigma(0) \leq C_{th}, \forall z \in Z, \forall p \in A_z, \forall i, \quad (10)
\]

which is quadratic in the state variables if the neighbor-crosstalk is ignored.

**IV. SIMULATION RESULTS**

To compare the performance of the RWA algorithms and the impact of physical impairments, three different networks are considered, shown in Fig. 1. We simulate the proposed optimal RWA, shortest path (SP) routing with an optimal wavelength assignment (WA), and shortest path routing with first-fit WA. They are referred to as Opt-RWA, Opt-WA, and SP-FF, respectively. Opt-WA uses the optimal RWA algorithm except that \(A_z\) contains only the SP route. We present results for transmission-impaired networks, labeled PHY, and for networks immune to physical impairments (no labeling).

Given R requests, the size of the set of possible requests generated is \((N \times (N - 1))^R\). It is impossible for us to solve the RWA problem for every point from this set because the RWA problem is NP-hard [5], [6]. Thus, one thousand sample
points are randomly chosen from the set of possible requests and the results are shown in Figs. 2 and 3.

We plot the mean and 90th percentile for the case of \( R = 12 \) in Fig. 2. The performance of Opt-RWA-PHY is on average one wavelength improvement over Opt-WA-PHY and SP-FF-PHY except in the tandem network. The gap between Opt-RWA and Opt-WA shows the improvement of using optimal routing on the network performance. Note that Opt-RWA performs the same as OPT-WA in the tandem network, because there is only one route for each s-d pair. Generally, Opt-WA-PHY is slightly better than SP-FF-PHY in any network, showing that the benefit of optimizing the WA is not as great as that from optimizing routing.

The impact of physical degradation is shown by comparing the solid lines to the adjacent dashed lines in Fig. 2. We observe the smallest gap occurs for Opt-RWA, followed by Opt-WA, and then SP-FF. Consequently, we conclude that optimal RWA algorithms can help reduce (yet not remove) the impact of physical degradations.

Fig. 3 shows the average wavelength resources needed for the three algorithms in different traffic loads (number of requests) for the 10-node ring network. As the traffic load increases, the gaps between the schemes without physical impairments and those with impairments increase. Their slopes show that optimal routing and optimal WA help more in large traffic cases. Note that WA is inconsequential if physical impairments are ignored but becomes important otherwise.

V. Conclusion

Optimal RWA algorithms, though computationally burdensome, are necessary to provide a formal description of the problem and to guide effective approximation methods. In this paper, we propose a mathematical formulation for the optimal RWA for networks with state dependent physical impairments. The optimal solution provides a performance lower bound for other QoT-aware RWA algorithms. Simulation results show that an optimal routing policy can effectively decrease the wavelengths resources needed and is more powerful than optimizing the WA if multiple paths are available. Improving the WA can further reduce the physical degradation, especially in tandem and other sparsely connected networks.

REFERENCES