

CSCI 124/224 - Discrete Structures II - Fall 2007
George Washington University

Homework 5: 30 points, 6%

due 6 December 2007, by 6 pm in instructor's mailbox, or in class.

Policy on collaboration: All examinations, papers, and other graded work products and assignments are to be completed in conformance with The George Washington University Code of Academic Integrity. **Unlike other HWs, you may NOT discuss this among yourselves. However, this HW, like others, is open book, so you may refer to your notes or other material.**

Any violations will be treated as violations of the Code of Academic Integrity.

There will be no partial credit

1. (2 points: no partial credit) Find the smallest positive value of x that satisfies the following equations:

$$x = 2 \pmod{4}$$

$$x = 3 \pmod{9}$$

2. (4 points: 1 per correct match) Match the following. Note that some phrases may match several, however, you will eliminate all possibilities except one match by ensuring that all phrases have a match.

- i. A vector in the span of $(1, 1, 1)$ and $(1, -1, 1)$
- ii. A vector in the span of $(1, 1, 1)$
- iii. $3 \times (1, 1, 1) + 4 \times (1, -1, 1)$
- iv. A vector not in the span of $(1, 1, 1)$ and $(1, -1, 1)$

(a) $(2, 2, 2)$

(b) $(1, 0, 0)$

(c) $(7, -1, 7)$

(d) $(2, 0, 2)$

3. (5 points: 1 per correct match) Match the following. Note that some phrases may match several, however, you will eliminate all possibilities except one match by ensuring that all phrases have a match.

- i. v_1 and v_2 are linearly independent. The vector $\mathbf{0}$ is $\mathbf{0} = a_1v_1 + a_2v_2$. What is a possible value of (a_1, a_2) ?
 - ii. $v_1 = (3, 3)$ and $v_2 = (1, 1)$. The vector $\mathbf{0}$ is $\mathbf{0} = a_1v_1 + a_2v_2$. What is a possible value of (a_1, a_2) ?
 - iii. What is a possible vector orthogonal to $(1, -1)$?
 - iv. Let $v_1 = (1, 0)$. If $\{v_1, v_2\}$ forms a basis for \mathbb{R}^2 , what is a possible value for v_2 ?
 - v. $v_1 = (1, -1)$, and $v_2 = (2, -2)$. v_3 lies in $\text{Span}(\{v_1, v_2\})$. What is a possible value for v_3 ?
 - (a) $(0, 3)$
 - (b) $(-2, 6)$
 - (c) $(5, 5)$
 - (d) $(5, -5)$
 - (e) $(0, 0)$
4. Say whether the following are True or False (12 points: +1 per correct answer, -1 per incorrect answer)
- (a) The vectors $v_1 = (1, 1)$ and $v_2 = (2, 2)$ are linearly independent
 - (b) The vectors $v_1 = (1, 1)$ and $v_2 = (1, -1)$ are linearly independent
 - (c) The vector $(a, 0, a)$ for any real value a is in the span of $(1, 1, 1)$ and $(1, -1, 1)$
 - (d) The vector $(0, a, 0)$ for any real value a is in the span of $(1, 1, 1)$ and $(1, -1, 1)$
 - (e) The vector $(a, a, 0)$ for any real value a is in the span of $(1, 1, 1)$ and $(1, -1, 1)$
 - (f) Suppose $v_1, v_2, v_3 \in \mathbb{R}^3$, and v_3 lies in the same plane as v_1 and v_2 . v_1, v_2 and v_3 are linearly independent.
 - (g) Suppose $v_1 = v_2 + v_3$. v_1, v_2 and v_3 are linearly independent.

- (h) Let $v_1 = (1, 1)$ and $v_2 = (2, 1)$. For any $v_3 = (x, y)$ for real x and y , there are unique coefficients a_1 and a_2 such that $v_3 = a_1v_1 + a_2v_2$.
- (i) Let $v_1 = (1, 1)$ and $v_2 = (2, 2)$. For any $v_3 = (x, y)$ for real x and y , there are unique coefficients a_1 and a_2 such that $v_3 = a_1v_1 + a_2v_2$.
- (j) Let $v_1 = (1, 1)$ and $v_2 = (2, 2)$. There are coefficients a_1 and a_2 such that $a_1v_1 + a_2v_2 = (0, 0)$.
- (k) Suppose $v_3 = 2v_1 + v_2 = v_2 + 2v_2$. v_1 and v_2 are linearly independent.
- (l) Two vectors in \mathbb{R}^2 that do not lie along the same line span \mathbb{R}^2 .
5. (3 points) f is a homomorphism from G_1 (the group of integers *mod* 4 with the operation \diamond , addition *mod* 4) to G_2 (the group of non-zero integers *mod* 5 with the operation \circ , multiplication *mod* 5). If $f(3) = 2$, what is $f(1)$?
6. (4 points) Is $v_4 = (0, 2, 1)$ in $\text{Span}(\{v_1 = (0, 1, 0), v_2 = (1, 1, 0), v_3 = (1, 2, 1)\})$? If so, what are the coefficients a_i , $i = 1, 2, 3$, in $v_4 = \sum a_iv_i$?
7. EXTRA CREDIT (1 point). A little learning is a dangerous thing. Drink deep of the fountain of knowledge. True/False?