CSCI 124/224 - Discrete Structures II - Fall 2007
George Washington University

Homework 1: 75 points

due 13 September 2007, by 6 pm in instructor's mailbox, or in class.

Policy on collaboration: All examinations, papers, and other graded work products and assignments are to be completed in conformance with The George Washington University Code of Academic Integrity. You may discuss HWs among yourselves, and work on them in groups. However, each student is expected to write his or her own HW out independently; you may not copy one another's assignments, even in part. You may not collaborate with others on the quizzes, tests or final.

You are expected to cite all your sources in any written work that is not closed book: papers, books, web sites, discussions with others - faculty, friends, students. For example, if, in a group, one student has a major idea that leads to a solution to a HW problem, all other students in the group should cite this student.

Any violations will be treated as violations of the Code of Academic Integrity.

1 (10 points) Show that, if a and b are integers such that $a|b$, then $a^k|b^k$ for every positive integer k.

2 (10 points) Use mathematical induction to show that $2^n \leq n! \quad \forall n \geq 4$.

3 (10 points) The sum of the first n odd numbers is n^2. Show this by using the formula, derived in class, for the sum of the first n numbers. Do not use mathematical induction directly.

3 (10 points) Use mathematical induction to show that $\sum_{i=1}^{n} \frac{1}{2^i} = 1 - \frac{1}{2^n}$ for all $n \geq 1$.

4 (10 points) Show that, if a and b are integers such that $a \equiv b \ (mod\ m)$, then $a^k \equiv b^k \ (mod\ m)$ for every positive integer k.

5 (10 points) Show that, if a is an even integer, $a^2 \equiv 0 \ (mod 4)$, and, if a is odd, $a^2 \equiv 1 \ (mod 4)$.

6 (15 points) Suppose that x, a and b are positive integers such that $a|b$. Suppose that $x\ rem\ a = x_a$ and $x\ rem\ b = x_b$. Show that $x_a = x_b\ rem\ a$.