CSCI 124 - Discrete Structures II - Spring 2006
George Washington University

Linear Equations Exercises

1. Evaluate the determinant of the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 1 \\ 1 & -1 & -1 & 2 \\ 1 & 1 & -1 & 2 \end{bmatrix}$$

2. Use Cramer's rule to solve the following set of equations.

$$\begin{array}{ccccccccc} w & + & x & + & y & - & z & = & 2 \\ w & + & 2x & + & y & + & z & = & 5 \\ w & - & x & - & y & + & 2z & = & 1 \\ w & + & x & - & y & + & 2z & = & 3 \end{array}$$

(You can use your answer to 1 here.)

3. Use Gauss-Jordan elimination to determine the inverses of the following matrices:

a.

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 1 \\ -1 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & -4 \\ 3 & 10 & -9 \\ 5 & -10 & 10 \end{bmatrix}$$

Answers

1. 8

2. $x = y = z = w = 1$

Solutions

1.

$$\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 1 \\ 1 & -1 & -1 & 2 \\ 1 & 1 & -1 & 2 \end{bmatrix}$$

Minor of first term, 1:

$$\begin{vmatrix} 2 & 1 & 1 \\ -1 & -1 & 2 \\ 1 & -1 & 2 \end{vmatrix}$$

$2(-2+2) - 1(-2-2) + 1(1+1) = 6$

Minor of second term, 1:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 1 & -1 & 2 \end{vmatrix}$$

$1(-2+2) - 1(2-2) + 1(-1+1) = 0$

Minor of third term, 1:

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & 2 \\ 1 & 1 & 2 \end{vmatrix}$$

$1(-2-2) - 2(2-2) + 1(1+1) = -2$

Minor of fourth term, -1:

$$\begin{vmatrix} 1 & 2 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix}$$

$1(1+1) - 2(-1+1) + 1(1+1) = 4$

Matrix determinant is:$1(6) - 1(0) + 1(-2) + 1(4) = 8$.

2.

$$\begin{array}{ccccccccccc} w & + & x & + & y & - & z & = & 2 \\ w & + & 2x & + & y & + & z & = & 5 \\ w & - & x & - & y & + & 2z & = & 1 \\ w & + & x & - & y & + & 2z & = & 3 \end{array}$$

(You can use your answer to 1e here.)

$$w = \frac{\begin{vmatrix} 2 & 1 & 1 & -1 \\ 5 & 2 & 1 & 1 \\ 1 & -1 & -1 & 2 \\ 3 & 1 & -1 & 2 \end{vmatrix}}{8}$$

(Denominator is answer to 1e.)

$$= \frac{2\{2(-2+2) - 1(-2-2) + 1(1+1)\} - 1\{5(-2+2) - 1(2-6) + 1(-1+3)\}}{8}$$

$$= + \frac{1\{5(-2-2) - 2(2-6) + 1(1+3)\} + 1\{5(1+1) - 2(-1+3) + 1(1+3)\}}{8}$$

$$= \frac{2\{4+2\} - 1\{4+2\} + 1\{-20+8+4\} + 1\{10-4+4\}}{8}$$

$$= \frac{12 - 6 - 8 + 10}{8} = 1$$

$$x = \frac{\begin{vmatrix} 1 & 2 & 1 & -1 \\ 1 & 5 & 1 & 1 \\ 1 & 1 & -1 & 2 \\ 1 & 3 & -1 & 2 \end{vmatrix}}{8}$$

$$= \frac{1\{5(-2+2) - 1(2-6) + 1(-1+3)\} - 2\{1(-2+2) - 1(2-2) + 1(-1+1)\}}{8}$$

$$= + \frac{1\{1(2-6) - 5(2-2) + 1(3-1)\} + 1\{1(-1+3) - 5(-1+1) + 1(3-1)\}}{8}$$

$$= \frac{1\{4+2\} - 2\{0\} + 1\{-4+2\} + 1\{2+2\}}{8}$$

$$= \frac{6 - 2 + 4}{8} = 1$$

$$y = \frac{\begin{vmatrix} 1 & 1 & 2 & -1 \\ 1 & 2 & 5 & 1 \\ 1 & -1 & 1 & 2 \\ 1 & 1 & 3 & 2 \end{vmatrix}}{8}$$

$$= \frac{1\{2(2-6) - 5(-2-2) + 1(-3-1)\} - 1\{1(2-6) - 5(2-2) + 1(3-1)\}}{8}$$

$$= + \frac{2\{1(-2-2) - 2(2-2) + 1(1+1)\} + 1\{1(-3-1) - 2(3-1) + 5(1+1)\}}{8}$$

$$= \frac{1\{-8+20-4\} - 1\{-4+2\} + 2\{-4+2\} + 1\{-4-4+10\}}{8}$$

$$= \frac{8+2-4+2}{8} = 1$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 5 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 3 \end{vmatrix}}{8}$$

$$= \frac{1\{2(-3+1) - 1(-3-1) + 5(1+1)\} - 1\{1(-3+1) - 1(3-1) + 5(-1+1)\}}{8}$$

$$= + \frac{1\{1(-3-1) - 2(3-1) + 5(1+1)\} - 2\{1(1+1) - 2(-1+1) + 1(1+1)\}}{8}$$

$$= \frac{1\{-4+4+10\} - 1\{-2-2\} + 1\{-4-4+10\} - 2\{2+2\}}{8}$$

$$= \frac{10+4+2-8}{8} = 1$$

Solution is $x = y = z = w = 1$