Find the modular inverse of the following numbers using the euclidean algorithm. You may use a calculator to compute remainders.

1. \(12^{-1} \mod 25\)

Answer:

\[
\begin{array}{c|c|c|c|c}
 a & b & q & s & t \\
25 & 12 & 2 & 1 & -2 \\
12 & 1 & 12 & 0 & 1 \\
1 & 0 & & & \\
\end{array}
\]

\[25(1) - (12)(2) = 1 \Rightarrow 12^{-1} \mod 25 = -2 \mod 25 = 23 \mod 25\]

2. \(17^{-1} \mod 20\)

Answer:

\[
\begin{array}{c|c|c|c|c}
 a & b & q & s & t \\
20 & 17 & 1 & 6 & -7 \\
17 & 3 & 5 & -1 & 6 \\
3 & 2 & 1 & 1 & -1 \\
2 & 1 & 2 & 0 & 1 \\
1 & 0 & & & \\
\end{array}
\]

\[20(6) + 17(-7) = 1 \Rightarrow -7 = 17^{-1} \mod 20 = 13.\]

3. \(9^{-1} \mod 25\)

Answer:

\[
\begin{array}{c|c|c|c|c}
 a & b & q & s & t \\
25 & 9 & 2 & 4 & -11 \\
9 & 7 & 1 & -3 & 4 \\
7 & 2 & 3 & 1 & -3 \\
2 & 1 & 2 & 0 & 1 \\
1 & 0 & & & \\
\end{array}
\]

\[25(4) + 9(-11) = 1 \Rightarrow -11 = 9^{-1} \mod 25 = 14.\]

4. \(15^{-1} \mod 32\)

Answer:

\[
\begin{array}{c|c|c|c|c}
 a & b & q & s & t \\
32 & 15 & 2 & -7 & 15 \\
15 & 2 & 7 & 1 & -7 \\
2 & 1 & 2 & 0 & 1 \\
1 & 0 & & & \\
\end{array}
\]
32(−7) + 15(15) = 1 ⇒ 15 = 15\text{−1}\text{ MOD 32}.

5. 3\text{−1} \text{ mod 32}

Answer:

\[
\begin{array}{c|c|c|c|c}
 a & b & q & s & t \\
\hline
 32 & 3 & 10 & -1 & 11 \\
 3 & 2 & 1 & 1 & -1 \\
 2 & 1 & 2 & 0 & 1 \\
 1 & 0 & & & \\
\end{array}
\]

32(−1) + 3(11) = 1 ⇒ 11 = 3\text{−1}\text{ MOD 32}. 