CSCI 124: Discrete Structures II: Discussion Session Questions: Inner Product

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Consider the three vectors:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_n \end{bmatrix} \qquad \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ \cdots \\ z_n \end{bmatrix}$$

Show that

1. The inner product is symmetric, i.e.:

$$\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{y}, \mathbf{x} \rangle$$

2. Also show that it is linear: a

$$\langle \mathbf{x}, \mathbf{y} + \mathbf{z} \rangle = \langle \mathbf{x}, \mathbf{y} \rangle + \langle \mathbf{x}, \mathbf{z} \rangle$$

b

$$\langle \mathbf{x}, c\mathbf{y} \rangle = c\langle \mathbf{x}, \mathbf{y} \rangle$$

3. From 1 and 2a, show that:

$$\langle \mathbf{x}, \mathbf{y} + \mathbf{z} \rangle = \langle \mathbf{y}, \mathbf{x} \rangle + \langle \mathbf{z}, \mathbf{x} \rangle$$

4. Suppose $\{\mathbf{b}_i\}_{i=1}^n$ is a basis for a vector space V and that $\{\mathbf{b}_i\}_{i=1}^n$ is also an orthonormal set. Show that any vector $\mathbf{x} \in V$ can be expressed as

$$\mathbf{x} = \sum_{i=1}^{n} \langle \mathbf{x}, \mathbf{b}_i \rangle \mathbf{b}_i$$

Hint: Suppose $\mathbf{x} = \sum_{i=1}^n c_i \mathbf{b}_i$ (because $\{\mathbf{b}_i\}_{i=1}^n$ is a basis for V, we know this is possible for all $\mathbf{x} \in V$. Show that $\langle \mathbf{x}, \mathbf{b}_i \rangle = c_i$ using 3 and 2b.

5. Use 4 to find c_1, c_2, c_3, c_4 when $\mathbf{x} = \sum_{i=1}^4 c_i \mathbf{b}_i$ for:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 9 \\ 7 \\ 2 \end{bmatrix} \qquad \mathbf{b}_1 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \qquad \mathbf{b}_2 = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \qquad \mathbf{b}_3 = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \qquad \mathbf{b}_4 = \frac{1}{2}\begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$