

CSCI 124

Discrete Structures II: Convolution and Polynomial Multiplication

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Consider the multiplication of two polynomials:

$$a(x) = \sum_{i=0}^{n_a} a_i x^i$$

and

$$b(x) = \sum_{i=0}^{n_b} b_i x^i$$

$$a(x)b(x) = \left(\sum_{i=0}^{n_a} a_i x^i\right)\left(\sum_{i=0}^{n_b} b_i x^i\right) = \sum_{j=0}^{n_a} a_j x^j \sum_{i=0}^{n_b} b_i x^i = \sum_{j=0}^{n_a} \sum_{i=0}^{n_b} a_j b_i x^{i+j}$$

For example:

$$a(x) = 3x^2 + 5x - 1$$

and

$$b(x) = 2x^2 - 3x + 2$$

$$a(x)b(x) = (3x^2 + 5x - 1)(2x^2 - 3x + 2) = 3x^2(2x^2 - 3x + 2) + 5x(2x^2 - 3x + 2) - 1(2x^2 - 3x + 2) = 6x^4 + x^3 - 11x^2 + 13x - 2$$

In general, the coefficient of x^k in the product polynomial will consist of

$$c_k = \sum_{j=0}^{n_a} a_j b_{k-j}$$

The convolution of two sequences a_i and b_i is c_i , denoted $a * b$:

$$c_i = \sum_{j=0}^{n_a} a_j b_{i-j}$$

Find the convolution of the following sequences:

1. Find $a_i * b_i$:

$$a_i = \begin{cases} 0 & i \leq -2 \\ 1 & -2 < i < 2 \\ 2 & 2 \leq i \leq 4 \\ 0 & i > 4 \end{cases} \quad b_i = \begin{cases} 0 & i \leq -2 \\ 1 & -2 < i < 2 \\ 0 & i \geq 2 \end{cases}$$

2. Find $a_i * b_i$:

$$a_i = b_i = \begin{cases} 0 & i < -2 \\ 1 & -2 \leq i \leq 2 \\ 0 & i > 2 \end{cases}$$

3. Find $a_i * x_i$ for any sequence x_i

$$a_i = \begin{cases} 0 & i \leq -1 \\ 1 & i = 0 \\ 0 & i \geq 1 \end{cases}$$