CSCI 124 - Discrete Structures II - Fall 2010
The George Washington University

Homework 5: 100 points
due 10 December 2010, 6 pm, in TA's mailbox

Policy on collaboration: All examinations, papers, and other graded work products and assignments are to be completed in conformance with The George Washington University Code of Academic Integrity. You may discuss HWs among yourselves, and work on them in groups. However, each student is expected to write his or her own HW out independently; you may not copy one another's assignments, even in part. You may not collaborate with others on the quizzes, tests or final.

You are expected to cite all your sources in any written work that is not closed book: papers, books, web sites, discussions with others - faculty, friends, students. For example, if, in a group, one student has a major idea that leads to a solution to a HW problem, all other students in the group should cite this student.

Any violations will be treated as violations of the Code of Academic Integrity.

Acknowledgements: some of these are from the book on Linear Algebra by Strang.

1. Construct:

(a) (10 points) a subset of the three-dimensional vector space over $\mathbb{R}$ that is closed under vector addition and subtraction, but not under scalar multiplication. Demonstrate why it is closed under vector addition and subtraction for general elements in the set, and provide an example that shows that it is not closed under scalar multiplication. **Note that an example of a subspace of $\mathbb{R}^2$ is not acceptable**.

(b) (10 points) a subset of the three-dimensional vector space over $\mathbb{R}$ that is closed under scalar multiplication, but not under vector addition. Demonstrate why it is closed under scalar multiplication for general elements in the set and in $\mathbb{R}$, and provide an example that shows that it is not closed under vector addition. **Note that an example of a subspace of $\mathbb{R}^2$ is not acceptable**.

2. Which of the following subsets of $\mathbb{R}$ are subspaces (support your claims with proofs for general elements if you claim it is a subspace, and with examples that show it does not satisfy the properties of a subspace if you claim it is not. You need only consider the two properties of a subspace: closure with respect to vector addition and scalar multiplication). For those that are subspaces, provide a basis (i.e. a set of linearly independent vectors that span the space). You need not prove that the set is a basis. You may assume that $b_1, b_2, b_2 \in \mathbb{R}$

a. (7 points) The set of vectors $b = (b_1, b_2)$ with $b_1 = 0$.

b. (5 points) The set of vectors $b = (b_1, b_2, b_3)$ with first component $b_1 = c \neq 0$.

c. (6 points) The vectors $b = (b_1, b_2, b_3)$ with $b_1 \times b_2 = 0$ (that is, the product of b_1 and b_2 is zero).

d. (3 points) The solitary vector $b = (1, 1, 1)$

e. (7 points) All linear combinations of two given vectors, $x = (3, 0, 9)$ and $y = (1, 0, 1)$

f. (7 points) The vectors $b = (b_1, b_2, b_3)$ that satisfy $2b_3 - 5b_2 + 6b_1 = 9$.

g. (7 points) The vectors $b = (b_1, b_2, b_3)$ that satisfy $2b_3 - 5b_2 + 6b_1 = 0$.

h. (7 points) The vectors $b = (b_1, b_2)$ that satisfy $b_1 + b_2 = 0$.

3. a. (7 points) Use the Gauss-Jordan technique for inverses to determine the inverse of the $n \times n$ matrix with the values c (such that $c \in \mathbb{R}$) on the anti-diagonal and zeroes elsewhere:

$$\mathbf{A}_n = \begin{bmatrix} 0 & 0 & . & . & . & . & . & c \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & . & . & c & . & . & 0 \\ .. & . & . & . & . & . & . & . \\ & . & . & . & . & . & . & . \\ 0 & c & . & . & . & . & . & 0 \\ c & 0 & . & . & . & . & . & 0 \end{bmatrix}$$

b. (2 points) For what values of c is the matrix above not invertible? Why?

4. Using the results above, give examples of 2×2 matrices A and B such that:

(i) (3 points) $A + B$ is not invertible though A and B both are

(ii) (3 points) A, B, and $A + B$ are all invertible

5. Find the inverses of the following matrices using Gauss-Jordan elimination:

a. (8 points)

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

b. (8 points) Note that $c \in \mathbb{R}$ such that $c \neq 0$:

$$B = \begin{bmatrix} 0 & 0 & c \\ c & 0 & 0 \\ 0 & c & 0 \end{bmatrix}$$

using either Cramer's rule or Gauss-Jordan elimination. Show all steps.