

CSCI 124 - Discrete Structures II - Fall 2009
George Washington University

Homework 1: 100 points

due 25 September 2009, by 6 pm in TA's mailbox.

Policy on collaboration: All examinations, papers, and other graded work products and assignments are to be completed in conformance with The George Washington University Code of Academic Integrity. You may discuss HWs among yourselves, and work on them in groups. However, each student is expected to write his or her own HW out independently; you may not copy one another's assignments, even in part. You may not collaborate with others on the quizzes, tests or final.

You are expected to cite all your sources in any written work that is not closed book: papers, books, web sites, discussions with others - faculty, friends, students. For example, if, in a group, one student has a major idea that leads to a solution to a HW problem, all other students in the group should cite this student.

Any violations will be treated as violations of the Code of Academic Integrity.

Acknowledgement: Several of these problems are from the textbook, or influenced by problems from the textbook.

1 (12 points) Show that, if a and b are integers such that $a|b$, then (i) $na|nb \forall n \in \mathbb{Z}$ and (ii) $a^k|b^k$ for every positive integer k .

2 (15 points) Suppose $a, d \in \mathbb{Z}$. Determine a formula for

$$a + (a + d) + (a + 2d) + (a + 3d) + \dots + (a + (n - 1)d)$$

for $n \in \mathbb{Z}^+$. Prove the correctness of the formula using mathematical induction.

3 (10 points) Suppose you are given integer m , such that $m > 1$. What is the sum of the first n positive integers x such that $x \text{ rem } m = 1$? Obtain the result by using the formula, derived in class, for the sum of the first n numbers. Do not use mathematical induction directly. Your formula will be a function of n and m .

4 (13 points) Show that, if a and b are integers such that $a \equiv b \pmod{m}$, then (i) $na \equiv nb \pmod{m} \forall n \in \mathbb{Z}$ and (ii) $a^k \equiv b^k \pmod{m}$ for every positive integer k .

5A. (15 points) Suppose that c, a and b are positive integers such that $a|b$. Suppose that $c \text{ rem } a = c_a$ and $c \text{ rem } b = c_b$. Show that $c_a = c_b \text{ rem } a$.

5B. (10 points) Suppose $m, n \in \mathbb{Z}$ such that $m = 2n$. Show that, if a is such that $a \text{ rem } n = 1$, then $a^2 \equiv 1 \pmod{m}$, or $a^2 \equiv n + 1 \pmod{m}$.

6. (10 points) Consider the set $\mathcal{G} = \{0, 1, 2, \dots, m - 1\}$. Let \diamond be the operation $a \diamond b = a + b + 1 \pmod{m}$, defined for all $a, b \in \mathcal{G}$. Is (\mathcal{G}, \diamond) a group? Why or why not?

7. (15 points) Let $\mathcal{R} = \mathbb{R}^+$, the set of positive real numbers. Let \oplus be an operation on \mathcal{R} , defined by $a \oplus b = ab$ for all $a, b \in \mathcal{R}$. Let $a \odot b = a^{\log_2 b}$. Is $(\mathcal{R}, \oplus, \odot)$ a ring? Why or why not?

8. (Extra Credit: 5 points) For $n \in \mathbb{Z}^+$ prove each of the following by mathematical induction: (a) $5|(n^5 - n)$ and (b) $6|(n^3 + 5n)$.