

**CSCI 124 - Discrete Structures II - Fall 2010**  
**George Washington University**

**Homework 1: 100 points**

due 24 September 2010, by 6 pm in TA's mailbox.

**Policy on collaboration:** All examinations, papers, and other graded work products and assignments are to be completed in conformance with The George Washington University Code of Academic Integrity. You may discuss HWs among yourselves, and work on them in groups. However, each student is expected to write his or her own HW out independently; you may not copy one another's assignments, even in part. You may not collaborate with others on the quizzes, tests or final.

You are expected to cite all your sources in any written work that is not closed book: papers, books, web sites, discussions with others - faculty, friends, students. For example, if, in a group, one student has a major idea that leads to a solution to a HW problem, all other students in the group should cite this student.

*Any violations will be treated as violations of the Code of Academic Integrity.*

**Acknowledgement:** Several of these problems are from the books by Koshy, Grimaldi and Rosen, or influenced by problems from these books.

1 (10 points) Show that, if  $a, b, c \in \mathbb{Z}$  such that  $a|b$  and  $b|c$ , then (i)  $na|nc \forall n \in \mathbb{Z}$  and (ii)  $a^k|c^k$  for every positive integer  $k$ .

2 (20 points) Show that, if  $a, b, c, d \in \mathbb{Z}$  such that  $a \equiv b \pmod{m}$  and  $c \equiv d \pmod{m}$ , then (i)  $na \equiv nb \pmod{m} \forall n \in \mathbb{Z}$  (ii)  $a^k \equiv b^k \pmod{m}$  for every positive integer  $k$  and (iii)  $n(a - c)^k \equiv n(b - d)^k \pmod{m} \forall n \in \mathbb{Z}$  and for every positive integer  $k$ .

3 (10 points) Suppose  $m, n$  are positive integers such that  $m|n$  and  $m \neq n$ . Further, consider a fixed  $c \in \mathbb{Z}$  such that  $0 \leq c < m$ . Suppose  $y \in \mathbb{Z}$  such that  $0 \leq y < n$  and  $y \equiv c \pmod{m}$ . What are *all* possible values of  $y$ ?

4. (10 points) Consider the set  $\mathcal{G} = \{0, 1, 2, \dots, m-1\}$ . Let  $c \in \mathbb{Z}$ . Let  $\diamond$  be the operation  $a \diamond b = a + b + c \pmod{m}$ , defined for all  $a, b \in \mathcal{G}$ . Is  $(\mathcal{G}, \diamond)$  a group? Why or why not?

5. (15 points) Let  $\mathcal{R} = \mathbb{R}^+$ , the set of positive real numbers. Let  $\oplus$  be an operation on  $\mathcal{R}$ , defined by  $a \oplus b = ab$  for all  $a, b \in \mathcal{R}$ . Let  $a \odot b = a^{\log_2 b}$ . Is  $(\mathcal{R}, \oplus, \odot)$  a ring? Why or why not?

6. (10 points) Show that, if  $a$  is an odd integer,  $a^2 \equiv 1 \pmod{8}$ .

7. (15 points) Use mathematical induction and results shown in class to prove that, if  $a_1, a_2, \dots, a_n \in \mathbb{Z}$ ,  $b_1, b_2, \dots, b_n \in \mathbb{Z}$  and  $m$  a positive integer, and, further, that  $a_i \equiv b_i \forall i$ ,

(a)  $\sum_{i=1}^n a_i \equiv \sum_{i=1}^n b_i \pmod{m}$

(b)  $\prod_{i=1}^n a_i \equiv \prod_{i=1}^n b_i \pmod{m}$

8 (12 points). Show the following formally:

- a. The sum of an even integer and an odd integer is odd.
- b. The product of an even integer and an odd integer is odd.
- c. The square of an integer is odd if and only if it is odd.