Solution Question 8.18

Assess the probability that you will be hospitalized for more than one day during the upcoming year.

$$Pr(Hospital) = Pr(Hospital|Accident)Pr(Accident) + \\ Pr(Hospital|No\,Accident)Pr(No\,Accident)$$

I would asses: $Pr(Hospital|Accident) = 1$ (I drive a motorcycle!)

When I assess: $Pr(Hospital|No\,Accident)$ I only need to consider my general health condition and possibly other external events, for example getting injured while exercising.

$Pr(Accident)$: This could be assessed from accident data using a similar driver profile as mine (possibly age dependent).

$Pr(No\,Accident) = 1 - Pr(Accident)$

Solution Question 8.20

After observing a long run of red on a roulette wheel, many gamblers believe that black is bound to occur. Such a belief is often called

Gambler's Fallacy

because the roulette wheel has no memory. Which probability assessment heuristic is at work in the gambler's fallacy? Explain

We know that: $Pr(Red) = 0.50$ and $Pr(Black) = 0.50$. Hence, we feel that in a string of sequences, these probabilies should be closely approximated (even when this string is relatively small). This is a form of the representative bias in the sense that one feels that any finite observed sequence should be representative of these probabilities.

Of course, the history of a series of occurences on the roulette wheel does not affect the next occurence at all. (Recall, each trial experiment is independent from the previous one).

Solution Question 8.25

It is not necessary to have someone else set up a series of bets against you in order for incoherence to take its toll. It is conceivable that one inadvertently get one-self in a no-win situation through inattention to certain details and the resulting incoherence, as the following problem shows.

Suppose that an executive of a venture-capital investment firm is trying to decide how to allocate his funds amongst three different projects, each of which requires a $\$100,000$ investment. The projects are such that one of the three will definitly succeed, but is not possible for more than one to succeed. Looking at each project as an investment, the anticipated payoff is good, but not wonderfull. If a project succeeds, the payoff will be a net gain of $\$150,000$. Of course, if the project fails, he loses all of the money invested in that project. Because he feels as though he knows nothing about wheter a project will succeed or fails, he assigns a probability of 0.5 that each project will succeed, and he decided to invest in each project.

a. According to his assessed probabilities, what is the expected profit for each project:

$$E[Profit\,per\,Project] =$$
$$0.5 \times (\$150,000) + 0.5(-\$100,000) = \$25,000$$

Hence, the investor believes according to his assessed probabilities that he will have an average profit of $\$75,000$.

b. What are the possible outcome for the three investments, and how much will he make in each case?

Indication a succesfull project with (1) and a failed project with (0) we have the following possible outcomes:

Project 1	1	0	0
Project 2	0	1	0
Project 3	0	0	1

Hence we have for the overall associated payoffs:

Project 1	\$150000	-\$100000	-\$100000
Project 2	-\$100000	\$150000	-\$100000
Project 3	-\$100000	-\$100000	\$150000
Total	-\$50000	-\$50000	-\$50000

Conclusion, the investor is guaranteed to loose money.

c. Do you think he invested wisely? Can you explaun why he is in such a predicament? No, he should have assessed Pr(Project i Successfull$) = \frac{1}{3}$, $i = 1, 2, 3$.

d. "Knowing nothing" about the occurrence of a particular event does not mean it's probability of occurring equals 0.5.

This works well for the case of throwing a coin, but it did not work well for the previous example.

Solution Question 9.15

a. Define: $T \equiv$ Time between two car sales

Let: m = Average number of Car Sales per hour

Assumptions meet the assumptions of the Poisson Process (Also the subscripts E and P indicate exponential distribution and Poisson Distribution in this Question)

$m =$ 8.5 cars per 10 hours - 0.85 cars per hour

$$Pr(T \leq t) = 1 - e^{-m \cdot t}$$

Hence, $Pr(T > 2\, hours) = e^{-0.85 \cdot 2} = e^{-1.7} = 0.183$

b. Define: $X \equiv$ Number of Car Sales in t hours

Then: $$Pr(X = k) = \frac{(m \cdot t)^k}{k!} e^{-m \cdot t}$$

Let: $t = 2\, hours$ and: $k = 0$

Remember: $m =$ 8.5 cars per 10 hours - 0.85 cars per hour

It thus follows that: $$Pr(X = 0) = \frac{(0.85 \cdot 2)^0}{0!} e^{-0.85 \cdot 2} = e^{-1.7} = 0.183$$

Note that the results in question a and b should be the same since if the time until the first customer arrives is more than two hours it immediately follows that no customers have arrived in the first two hours and vice versa.

c. Here we let t be 10 hours (That is a full day)

$$Pr(Bonus = \$20) = Pr(X = 13) = \frac{(0.85 \cdot 10)^{13}}{13!} e^{-0.85 \cdot 10} = 0.04$$

(Table Page. 700)

$$Pr(Bonus = \$30) = Pr(X = 14) = \frac{(0.85 \cdot 10)^{14}}{14!} e^{-0.85 \cdot 10} = 0.024$$

(Table Page. 700)

$$Pr(Bonus = \$50) = Pr(X = 15) = \frac{(0.85 \cdot 10)^{15}}{15!} e^{-0.85 \cdot 10} = 0.014$$

(Table Page. 700)

$$Pr(Bonus = \$70) = Pr(X \geq 16) = 1 - Pr(X \leq 15)$$

$$= 1 - \sum_{k=0}^{15} \frac{(0.85 \cdot 10)^{k}}{k!} e^{-0.85 \cdot 10} = 1 - 0.986 = 0.014$$

(Table Page. 705)

$$E[Bonus] = 0.04 \times \$20 + 0.024 \times \$30 + 0.014 \times \$50 + 0.014 \times \$70$$
$$= \$3.20$$

d. We have four Saturdays and Four Sundays, thus a total of 8 trials. Purchases appear to be independent from one another. Define:

"Success"= Owner will have to pay the $20 bonus

Then we have

$$Pr(Success) = Pr(Bonus = \$20) = 0.04$$

Define:

$$Y = Number\,of\,Successes\,in\,8\,Days$$

Then:

$$Y \sim Bin(8, 0.04)$$

and:

$$Pr(Y = 2) = \binom{8}{2}(0.04)^2(1 - 0.04)^6 \approx 0.035 \text{ (Table Page. } 682\text{)}$$

$$Pr(Y = 2) = \binom{8}{2}(0.04)^2(1 - 0.04)^6 \approx 0.0343 \text{ (With Excel)}$$

Solution Question 9.21

Define: X = Number of breakdowns of old machines in $[0, t]$

where: t has the dimension "months"

Define: Y = Number of breakdowns of new machines in $[0, t]$

where: t has the dimension "months"

Then: $X \sim Poisson(m \cdot t)$ $\quad Y \sim Poisson(n \cdot t)$

Where: $m = 2.5\, per\, months$ and $\begin{cases} Pr(n = 1.5\, per\, month) = 50\% \\ Pr(n = 3\, per\, month) = 50\% \end{cases}$

Repair of a new machine costs \$170 and of an old machine \$150

a. Let t be 1 month (hence we are comparing machine on a month to month bases). We have for the expected repair cost the following:

$$E[X|m = 2.5, t = 1] = m \times t = 2.5\ machines$$
$$\Rightarrow E[Cost] = 2.5 \times \$150 = \$375 \text{ per month}$$

$$E[Y|n = 1.5, t = 1] = n \times t = 1.5\ machines$$
$$E[Y|n = 3, t = 1] = n \times t = 3\ machines$$
$$\text{Hence, } E[Y|t = 1] = 50\% \times 1.5 + 50\% \times 3 = 2.25$$

$$\Rightarrow E[Cost] = 2.25 \times \$170 = \$382.50 \text{ per month}$$

Hence, the old machines cost less on a month to month basis

b. Data = $(6,[0,3))$ (That is 6 breakdowns in 3 months)

$$Pr(n=1.5|Data) = \frac{Pr(Data|n=1.5)Pr(n=1.5)}{Pr(Data|n=1.5)Pr(n=1.5)+Pr(Data|n=3)Pr(n=3)}$$

$$Pr(Data|n=1.5) = Pr(Y=6|n=1.5,\ t=3) = \frac{(1.5\times 3)^6}{6!}e^{-1.5\times 3} = 0.128$$

(Table Page 698)

$$Pr(Data|n=3) = Pr(Y=6|n=3,\ t=3) = \frac{(3\times 3)^6}{6!}e^{-3\times 3} = 0.091$$

(Table Page 700)

$$Pr(n=1.5|Data) = \frac{0.128\times 0.5}{0.128\times 0.5+0.091\times 0.5} = 0.5845$$

c. Let t be 1 month (hence we are comparing machine on a month to month bases). We have for the expected repair cost the following

$$E[X|m = 2.5, t = 1] = m \times t = 2.5\ machines$$
$$\Rightarrow E[Cost] = 2.5 \times \$150 = \$375 \text{ per month}$$

$$\begin{cases} Pr(n = 1.5\, per\, month|Data) & = 58.45\% \\ Pr(n = 3\, per\, month|Data) & = 41.55\% \end{cases}$$

$$E[\text{Y}|n = 1.5, t = 1] = n \times t = 1.5\ machines$$
$$E[\text{Y}|n = 3, t = 1] = n \times t = 3\ machines$$
$$\text{Hence, E[Y|}t = 1,\ Data] = 58.45\% \times 1.5 +\ 41.55\% \times 3 = 2.12$$

$$\Rightarrow E[Cost] = 2.12 \times \$170 = \$360.96 \text{ per month}$$

Hence, based on the data you would now prefer the new machines

QUESTION 9.28

$$T = Length\,of\,Strike\,in\,days$$

$$T \sim Uniform[0, 10.5]$$

$$f(t|a,b) = \frac{1}{(b-a)},\, a = 0,\, b = 10.5$$

a.

$$Pr(T \leq 1) = \int_0^1 f(u|a=0, b=10.5)du = \frac{1-a}{b-a} = \frac{1}{10.5} \approx 0.095$$

b.

$$Pr(T \leq 6) = \int_0^6 f(u|a = 0, b = 10.5)du = \frac{6-a}{b-a} = \frac{6}{10.5}$$

c.

$$Pr(6 \leq T \leq 7) = \int_6^7 \frac{1}{b-a}du = \frac{7-6}{b-a} = \frac{1}{10.5}$$

d.

$$Pr(T \leq 7|T > 6) = \frac{Pr(T \leq 7, T > 6)}{Pr(T > 6)} =$$

$$\frac{Pr(6 \leq T \leq 7)}{1 - Pr(T \leq 6)} = \frac{\frac{1}{10.5}}{1 - \frac{6}{10.5}} \approx 0.22$$

In Question 9.12 use:

$$T = Length\, of\, Strike\, in\, days$$

$$f(t|\lambda) = \lambda exp(-\lambda t), t \geq 0$$

$$E[T] = 10 = \frac{1}{\lambda} \Leftrightarrow \lambda = \frac{1}{10}$$

$$Pr(T \leq 1) = \int_0^1 \lambda exp(-\lambda u)du = 1 - exp(-0.1) \approx 0.095$$

$\boldsymbol{b.}$

$$Pr(T \le 6) = \int_0^6 \lambda exp(-\lambda u)du = 1 - exp(-0.1 \times 6) \approx 0.45$$

$\boldsymbol{c.}$

$$Pr(6 \le T \le 7) = \int_6^7 \lambda exp(-\lambda u)du =$$

$$exp(-0.1 \times 7) - exp(-0.1 \times 6) \approx 0.05$$

$d.$

$$Pr(T \leq 7 | T > 6) = \frac{Pr(T \leq 7, T > 6)}{Pr(T > 6)} =$$

$$\frac{Pr(6 \leq T \leq 7)}{1 - Pr(T \leq 6)} = \frac{0.05}{1 - 0.45} \approx 0.095$$

Solution Question 9.31

a. Define: L = Width of a Post Card

Then: $L \sim Normal(\mu, \sigma)$ where $\mu = 5.9in$ and $\sigma = 0.0365\,in$

Pr(Card does not fit in Envelope)=

$$Pr(L > 5.975|\mu = 5.9, \sigma = 0.0365) =$$

$$1 - Pr(L \leq 5.975|\mu = 5.9, \sigma = 0.0365) =$$

$$1 - Pr(\frac{L-5.9}{0.0365} \leq \frac{5.975-5.9}{0.0365}|\mu = 5.9, \sigma = 0.0365) = 1 - Pr(Z \leq 2.05)$$

Where: $Z \sim Normal(0, 1)$

Utilizing the Table on Page 709:

$$Pr(L > 5.975 | \mu = 5.9, \sigma = 0.0365) = 1 - 0.9798 = 0.0202$$

b. Define: X = Number of Cards in a box of 20 that do not fit in envelope

Then: $$X \sim Binom(20, p = 0.02)$$

$$Pr(X \geq 2 | n = 20, p = 0.02) = 1 - Pr(X \leq 1 | n = 20, p = 0.02)$$
$$= 1 - 0.94 = 0.06$$ (Table Page 694)

Hence, there is 6% probability that 2 or more cards will not fit.