CS 211: Fall 2008. Homework 1 Solutions

Ques.1: Consider the process of improving the performance of a program by optimizing some of the code in the program and running it in an enhanced (i.e., optimized) form. Suppose that optimized instructions run 20 times faster than sequential.

- Derive the speedup equation when *x%* of the instructions are optimized.
- Determine the percentage of code that must be optimized to get a speedup of 2,5, and 10 respectively.
- If *25%* of the code cannot be optimized (due to the inherently sequential nature such as I/O etc.), then what is the maximum speedup you can achieve.

Ans: The speedup S is defined as $S = T_{seq}/T_{par}$ where T_{seq} is non-optimized sequential and T_{par} is optimized (parallel) time. For each sequential cycle, the optimized time is (1/20 = 0.05). If each sequential instruction takes 1 cycle, the sequential time for a program with N instructions is $T_{seq} = N$ cycles. The time for a program with *x%* of its instructions optimized is *(N*0.05*x + N*1*(1-x))= N(0..05x+(1-x)).*

Therefore speedup $S = N/N(0.05x + (1-x)) = 1/(1-0.95x)$. Use this equation to determine the values of x for $S=2,5,10$. Rewrite the above speedup equation to solve for x, the percentage of code optimized, to get $x= (100/95) (1 – 1/S)$. Next insert the desired values of *S* to solve for *x*.

- For S=2, we get $x =(100/95)(1 – ½) = 52.6\%$
- For S=5, we get $x = (100/95)(1 – 1/5) = 84.2\%$
- For S=10, we get $x = (100/95)(1- 1/10) = 94.7\%$

If 25% of the code cannot be optimized, then maximum value of $x = 0.75$. Substitute in the speedup equation to get maximum speedup $S = 1/(1 – 0.95*0.75) = 3.47$.

(Practice Problem) Ques.3: This question requires you to generalize Amdahl's law to the case when multiple enhancements are possible. Three enhancements with the following speedups are proposed for a new architecture:

$Speedup_1$=30, $Speedup_2$= 20, $Speedup_3$ = 15.

Only one enhancement is usable at a time (but multiple can be used over the entire application). If enhancements 1 and 2 are each usable for 25% of the time, what fraction of the time must enhancement 3 be used to achieve an overall speedup of 10 for the entire application ?

Ans: Amdahl's Law can be generalized to handle multiple enhancements. If only one enhancement can be used at a time during program execution, then

$$Speedup= 1/[1 - \sum_i FE_i + \sum_i FE_i/SE_i]$$

where *FEi* is the fraction of time that enhancement *i* can be used and *SEi* is the speedup of enhancement *i.* For a single enhancement the equation reduces to the familiar form of Amdahl's Law. With three enhancements we have

$$Speedup= 1/[1 – (FE_1+ FE_2 + FE_3) + (FE_1/SE_1) + (FE_2/SE_2) + (FE_3/SE_3)]$$

Substituting in the known quantities gives

$$Speedup= 1/ [1 – (0.25 + 0.25 + FE_3) + (0.25/30) + (0.25/20) + (FE_3/15)]$$

Solving the above equation for FE_3 gives $FE_3= 0.45$. Thus, the third enhancement must be usable *45%* of the time.