

CS 178 Homework 4 Solutions

Ques.1: Consider the following collection of relations and dependencies. For each relation(s) (a) determine the candidate keys, (b) state the strongest normal form that the relation is in (from 2NF to BCNF), and (c) if a relation is not in BCNF then decompose it into a collection of BCNF relations.

a. $R_1(A,C,B,D,E)$, $A \rightarrow B$, $C \rightarrow D$

First compute the keys for R_1 . The attributes A , C , E do not appear on right hand side of any functional dependency therefore they must be part of a key. So start by computing the close of the set $\{A,C,E\}$. Since $A \rightarrow B$, we have $\{ACE\}^+ = \{ACEB\}$. Since $C \rightarrow D$, we have $\{ACE\}^+ = \{ACEBD\}$ and therefore the key is $\{ACE\}$.

We have dependencies $A \rightarrow B$ and $C \rightarrow D$ which are partial dependencies (i.e., depends on part of the key) and the relation is not in second normal form.

Applying the BCNF decomposition algorithm, the non-BCNF dependency is $A \rightarrow B$, therefore create two relations (A,C,D,E) and (A,B) . The first relation is still not in BCNF since we have a non-BCNF dependency $C \rightarrow D$. Therefore decompose further into (A,C,E) and (C,D) . Now all relations are in BCNF and the final BCNF scheme is (A,C,E) , (C,D) , (A,B) .

b. $R_2(A,B,F)$, $AB \rightarrow F$, $B \rightarrow F$

First compute keys for R_2 . A,B do not appear on RHS of any dependency, so start by computing attribute set closure of $\{AB\}$. Since $AB \rightarrow F$, we have $\{AB\}^+ = \{ABF\}$ and therefore $\{AB\}$ is the key. Since we have $B \rightarrow F$, i.e., F is partially dependent on the key, the relation is not in second normal form. During BCNF decomposition, we have $B \rightarrow F$ as the non-BCNF relation therefore create new schema (A,B) (B,F) . Both are in BCNF. Note however that now we have lost the dependency $AB \rightarrow F$.

c. $R_3(A,B)$, $R_4(C,D,E,F)$ $A \rightarrow B$, $C \rightarrow D$, $D \rightarrow EF$

The key for R_3 is $\{A\}$ since we have $A \rightarrow B$ and $\{A\}^+ = \{AB\}$. The key for R_4 is $\{C\}$ since we have $\{C\}^+ = \{CDEF\}$. R_3 is in BCNF. But R_4 has a transitive dependency in $D \rightarrow EF$ and therefore it is not in 3NF (but it is in second normal form). Applying the BCNF decomposition on R_4 , we get $D \rightarrow EF$ is a non-BCNF dependency and we decompose R_4 into two tables (C,D) and (D,E,F) both of which are in BCNF. Therefore the final schema is (A,B) , (C,D) , (D,E,F) .

d. $R_5(A,B,C,D,E)$ with functional dependencies $D \rightarrow B$, $CE \rightarrow A$. The key for R_5 is $\{CDE\}$ since we have $\{CDE\}^+ = \{ABCDE\}$. R_5 is not in BCNF and not in 3NF (in fact it has partial dependencies and is not even in second normal form). Applying the BCNF decomposition, pick $D \rightarrow B$ which is not BCNF and decompose into (D,B)

and (ACDE). But (ACDE) is still not in BCNF since $CE \rightarrow A$ is not BCNF. Therefore decompose into (D,B), (A,C,E) and (D,C,E) which is now in BCNF.

e. R6(A,B,C,D,E) with functional dependencies $A \rightarrow E$, $BC \rightarrow A$, $DE \rightarrow B$. Since D and C do not appear on right hand side, all keys must contain C and D. The set {C,D} by itself is not a key since $\{C,D\}^+ \neq \{C,D\}$. However, all three element sets containing {C,D} are keys – i.e., {A,C,D} is a key, {B,C,D} is a key, and {C,D,E} is a key. So all attributes are prime attributes. Therefore the schema is in 3NF. However, the left hand side of the FDs is not a key therefore it is not in BCNF. One decomposition will start by removing $A \rightarrow E$ to form (A,E) and (A,B,C,D). But (A,B,C,D) is not in BCNF since BC is not the key. Therefore decompose further to get (A,E), (BCA) and (BCD). This is in BCNF since the only dependency in (BCD) is the trivial dependency $BCD \rightarrow BCD$.

Ques.2: Consider a database of ship voyages with the following attributes: S (ship name), T (type of ship), V (voyage identifier), C (cargo carried by one ship on one voyage), P (port) and D (day). We assume that a voyage consists of a sequence of events where one ship picks up a single cargo, and delivers it to a sequence of ports. A ship can visit only one port in a single day.

This description implies the following functional dependencies:

$S \rightarrow T$

$V \rightarrow SC$

$SD \rightarrow PV$

Determine the key(s) for this relation (assuming all attributes are in one table to begin with).

The set of attributes are: (S, T, V, C, D,P).

Examining the FD's, we see that D does not appear on the right hand side of any dependency, therefore it must be part of any key.

So the smallest set that can be a key is {D}. However, $D^+ = \{D\}$. So we examine two element sets. Picking {S,D}, we get $\{S,D\}^+ = \{S,D,T,P,V,C\}$ and therefore it is a key. Similarly, $\{V,D\}^+ = \{S,C,P,V,T,D\}$ and therefore it is a key. Sets {D,C}, {D,T} and {D,P} will not give us a key.

The non-BCNF dependencies are $S \rightarrow T$ and $V \rightarrow SC$

These are also non-3NF dependencies (both are partial dependencies).

A lossless join decomposition into BCNF is ST, VC, VS, VDP, another is ST, VSC, VDP. There is not one that preserves dependencies, because to be able to check $SD \rightarrow PV$ (which is $SD \rightarrow P$ and $SD \rightarrow V$) you need to keep them together.

ST, VC, SVPD is in 3NF

