CS212 June 23, 2009

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Homework 4

Due Date: June 30, 2009

Problem 1: (15 points)

Let G = (V,E) be the following undirected graph: V = $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, and

E = $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4), (4, 5), (5, 7), (4, 6), (6, 7), (8, 9), (8, 7),$
$(9, 7), (7, 10), (8, 10), (9, 10)\}$.

Use backtracking to color G with 4 colors only. Find 4 solutions.

Problem 2: (20 points)

a) Let $G = (V, E)$ be an undirected graph. An indpendent set of G is a subset of nodes of G such that no two nodes are adjacent. Write a backtracking algorithm that generates all the independent sets of k nodes of G.

b) Let $E = \{1, 2, 3, ..., n\}$, and let k be a positive even integer $< n$. Write a backtracking algorithms that generates all the k-element subsets of E such that each subset has $\frac{k}{2}$ even integers and $\frac{k}{2}$ odd integers. Your algorithm must not generate any subset more than once.

c) Write a backtracking algorithm to generate all the arrays $X[1..n]$ such that each $X[i] \in \{0, 1, 2, 3\}$ and for $1 < i < n$ we have $X[i] > X[i - 1]$ or $X[i] > X[i + 1]$ (the 'or' is not exclusive).

Problem 3: (15 points)

Apply the branch and bound algorithm of the minimum job assignment problem to the following matrix:

$$\begin{pmatrix} 3 & 7 & 4 \\ 2 & 3 & 5 \\ 3 & 10 & 10 \end{pmatrix}$$

showing the value of the approximate cost function of every node. Give the solution produced by the algorithm for this problem.

Problem 4: (25 points)

Suppose you have 3 processors labeled P1, P2, and P3, such that processor P1 is 4 times as fast as P2 and 2 times as fast as P3. Assume that you have n programs and that the i-th program takes t_i time units to execute on processor P1 (and hence it takes $4t_i$ time units to execute on processor P2 and $2t_i$ time units on P3). A *schedule* S of the n programs on the 3 processors is an assignment that assigns every program to one of the processors. That is, S is a mapping of $\{1, 2, ..., n\}$ to $\{1, 2, 3\}$ where $S(i) = j$ means that program i is assigned to processor Pj. Let T_1 be the time that processor P1 needs to carry out a schedule S, that is, T_1 is the sum of the times t_i of the programs assigned to processor P1. Define T_2 and T_3 similarly, that is, T_2 is the sum of the times $4t_i$ of the programs assigned to processor P2, and T_3 is the sum of the times $2t_i$ of the programs assigned to processor P3. The cost of a schedule S is $\max(T_1, T_2, T_3)$. An optimal schedule is a schedule of minimum cost.

a) Give a branch and bound algorithm to compute an optimal schedule.

b) Apply your algorithm on the following 5 programs of times $t_1 = 5$, $t_2 = 7$, $t_3 = 2$, $t_4 = 3$, $t_5 = 1$. You should show the tree and the value of the approximate cost function of every node. Also point out the resulting optimal schedule. Note: whenever you have a tie while choosing an E-node, break the tie by selecting the deepest E-node in the tree.

Problem 5: (25 points)

Let G be an n-node undirected graph where each edge has a positive weight. A *hamiltonian path* in G is a path where every node of G appears exactly once. The weight of the path is the sum of the weights of its edges.

a) Give a branch and bound algorithm to compute a minimum-weight hamiltonian path (in G) that starts at node 1.

b) Apply your algorithm on the following graph:
$V = \{1, 2, 3, 4, 5\}$ and $E = \{(1, 2), (1, 5), (2, 5), (3, 4), (3, 5), (5, 4)\}$ of weights 4, 1, 2, 1, 1, 2, respectively.

You should show the tree and the value of the approximate cost function of every node. Also point out the resulting hamiltonian path.