CS212 June 9, 2009

Youssef

Homework 1

Due Date: June 23, 2009

Problem 1: (16 points)

Let $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$ be 6 numbers accessible with the following probabilities: $p_1 = \frac{3}{20}, p_2 = \frac{2}{20}, p_3 = \frac{1}{20}, p_4 = \frac{2}{20}$, $p_5 = \frac{2}{20}$ and $p_6 = \frac{2}{20}$. Let q_0 be the probability of searching for an item $< a_1$, q_6 the probability of searching for an item $> a_6$, and q_i the probability of searching for an item between a_i and a_{i+1} for $i = 1, 2, 3, 4, 5$. Take $q_0 = \frac{2}{20}$ and $q_i = \frac{1}{20}$, for $i = 1, ..., 6$. You are to construct an optimal binary search tree.

a) Write down a table for the w_{ij}'s, and a table for the C_{ij}'s and their corresponding r_{ij}'s as defined in the algorithm for optimal binary search trees.

b) Contruct the optimal binary search tree.

Problem 2: (20 points)

Let $A[1..n]$ be a real array, and let k be an integer, $1 \leq k \leq n$. A *linear k-partition* of A is any sequence of subarrays of the form $A[1..n_1]$, $A[n_1 + 1..n_2]$, ... , $A[n_{k-1} + 1..n]$, for some n_1, n_2, ... , n_{k-1} where $1 \leq n_1 < n_2 < n_3 < ... < n_{k-1} < n$. The *weight* of this linear k-partition is defined to be

$$MAX(\sum_{i=1}^{n_1} A[i], \sum_{n_1+1}^{n_2} A[i], \ ... \ , \sum_{n_{k-1}+1}^{n} A[i]).$$

Give a dynamic programming algorithm to compute the minimum-weight k-partition of A. (Hint: Take S_j^l as the weight of the minimum-weight l-partition of the array $A[1..j]$, and develop a recurrence relation for S_j^l.) Analyze the time complexity of your algorithm.

Problem 3: (15 points)

Consider binary trees of n nodes labeled $1, 2, ..., n$ such that the preorder sequence of the nodes is: $1, 2, ..., n$. Let $X[1..n]$ be a permutation of $\{1, 2, ..., n\}$, that is, the sequence $X[1]$, $X[2]$, ... , $X[n]$ is a rearrangement of $1, 2, ..., n$. Write an algorithm that constructs the binary tree whose inorder traversal sequence is $X[1..n]$. Give the time complexity of your algorithm.

Problem 4: (15 points)

Let I_1, I_2, ... , I_n be n intervals in the real axis (an interval I is of the form $[a, b]$ and represents the set $\{x | \ a \leq x \leq b\}$). A *rotation* is a sequence of intervals I_{i_1}, I_{i_2}, ... , I_{i_k} such that (1) every two consecutive intervals overlap, and (1) the first interval overlaps with the last. Write an algorithm that deletes the minimum number of intervals from $I_1, I_2, ..., I_n$ so that the remaining intervals have no rotations.

Problem 5: (15 points)

Let G = (V,E) be the following undirected graph: V = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, and E = $\{(1, 3), (3, 6), (1, 6), (6, 5), (6, 4), (7, 4), (5, 10), (5, 11),$
$(7, 9), (8, 7), (9, 5), (10, 11), (4, 2), (2, 8), (4, 8)\}$.

a) Do a depth-first search on G showing the depth-first search tree and the backward edges.

b) Do a breadth-first search tree on G showing the breadth-first tree and the cross edges.

Problem 6: (15 points)

Characterize the graphs that have a depth-first tree and a breadth-first tree that are identical.