CS123 March 26, 2002
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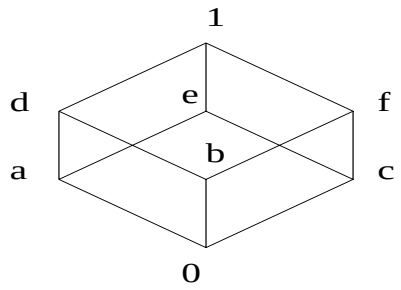
Homework 4
Due Date: April 23, 2002

Problem 1: (15 points)

Consider the Boolean algebra $(B, +, \bullet, ', 0, 1)$ represented by this Hasse diagram:



Give the tables of $+$, $\bullet$, and $'$.

Problem 2: (20 points)

For any integer $n > 1$ let D_n be the set of positive divisors of n. Define $\vee$, $\wedge$, and $'$ on D_n by $a \vee b = lcm(a, b)$ (that is, least common multiple of a and b), $a \wedge b = gcd(a, b)$ (that is, greatest common divisor of a and b), and $a' = n/a$. Define in D_n the relation $\preceq$: $a \preceq b$ if $a \vee b = b$.

a) Show that $a \preceq b$ if and only if a divides b.

b) Prove that $\preceq$ is a partial order relation in D_n.

c) Draw the Hasse diagrams of D_4, D_6, D_8 and D_{10}.

d) Which of the following sets is a Boolean algebra: D_4, D_6, D_8 and D_{10}? Explain why. For each Boolean algebra indicate what its 0 and 1 elements are.

Problem 3: (15 points)

Let $B = \{0, 1\}$ be a Boolean algebra and let $f: B^3 \to B$ be the Boolean function such that $f(0, 0, 0) = f(0, 0, 1) = f(1, 0, 1) = 1$ and $f(x, y, z) = 0$ for all other $(x, y, z) \in B^3$.

a) Write f in disjunctive normal form and in conjunctive normal form.

b) Give the truth table of f' (the complement of f).

c) Give f' in disjunctive normal form and in conjunctive normal form.

Problem 4: (20 points)

Minimize each of the following Boolean expressions using Karnaugh maps. Show the Karnaugh maps.

a) $xyz' + x'y + x'y'z'$

b) $x'z' + yz + y'z$

c) $xyw + xy'z'w + x'y'zw' + yzw' + xy'z$

d) $xz' + x'w'z' + xzw + x'y'w + y'zw + x'yw$

Problem 5: (15 points)

Let f and g be two Boolean functions of $x,\ y,\ z,\ w$ where $f(x,y,z,w) = 1$ if and only if at least two of $x,\ y,\ z,\ w$ have value 1, and $g(x,y,z,w) = 1$ if and only if at most two of $x,\ y,\ z,\ w$ have value 1.

a) Give the truth tables of f and g.

b) Minimize f and g using Karnaugh maps

Problem 6: (15 points)

Let $(B, +, \bullet, ', 0, 1)$ be a Boolean algebra. Define $\odot$ as follows: $x \odot y = (x \bullet y)'$.

a) Evaluate $x \odot 1$ and $x \odot 0$.

b) Express $x \bullet y$ using the operation $\odot$ but without using $+$, $\bullet$, or $'$.

c) Express $x + y$ using the operation $\odot$ but without using $+$, $\bullet$, or $'$.