

**CSCI 124 and CSCI 224 - Discrete Structures II**  
**Fast Exponentiation Practice Problems**

Compute the following exponentiations using the fast algorithm, square and multiply.

1.  $39^{215} \pmod{121}$

2.  $52^{21} \pmod{63}$

3.  $111^{67} \pmod{129}$

Solutions

1.  $39^{215} \pmod{121}$

$215 = 11010111$

$i$	$b_i$	$z_{first}$	$z_{second}$
7	1	1	39
6	1	69	29
5	0	115	115
4	1	36	73
3	0	5	5
2	1	25	7
1	1	49	96
0	1	20	54

$39^{215} \pmod{121} = 54$

2.  $52^{21} \pmod{63}$

$21 = 10101$

$i$	$b_i$	$z_{first}$	$z_{second}$
4	1	1	52
3	0	58	58
2	1	25	40
1	0	25	25
0	1	58	55

$52^{21} \pmod{63} = 55$

3.  $111^{67} \pmod{129}$

$67 = 1000011$

$i$	$b_i$	$z_{first}$	$z_{second}$
6	1	1	111
5	0	66	66
4	0	99	99
3	0	126	126
2	0	9	9
1	1	81	90
0	1	102	99

$111^{67} \pmod{129} = 99$